Experiment No : M8

Experiment Name: Steiner's theorem (parallel axis theorem)

Objective:

1. Determining the moment of inertia of a circular disk for various distances between the axis of rotation and the axis of symmetry.
2. Confirming Steiner's theorem (parallel axis theorem).

Keywords: Rotational motions, moment of inertia, parallel axis theorem, oscillation, period.

Theoretical Background:

The moment of inertia of an arbitrary rigid body whose mass elements Δm_i have the distances r_i from the axis of rotation A is

$$I^A = \sum_i \Delta m_i r_i^2 \tag{8.1}$$

If the axis of rotation *does not* pass through the centre of mass (CM) of the body, application of Eq. 8.1 leads to an involved calculation. Often it is easier to calculate the moment of inertia I_{CM} with respect to the axis *S*, which is parallel to the axis of rotation (*A*) and passes through the centre of mass of the body.

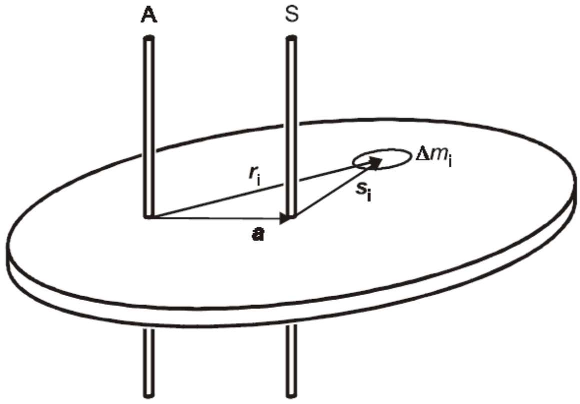

Figure 8.1: Schematic illustration referring to the derivation of Steiner's theorem (parallel axis theorem)

For deriving the relation between I^A and I_{CM}, the plane perpendicular to the axis of rotation where the respective mass element Δm_i is located is considered (see Fig. 8.1). In this plane, the vector ***a*** points from the axis of rotation to the centre-of-mass axis, the vector r_i points from the axis of rotation to the mass element Δm_i, and the vector s_i points from the centre-of-mass axis to the mass element. Thus

$$r_i = a + s_i \tag{8.2}$$

and the squares of the distances in Eq. 8.1 are

$$r_i^2 = (a + s_i)^2 = a^2 + 2.a.s_i + s_i^2 \tag{8.3}$$

Therefore the sum in Eq.8.1 can be split into three terms:

$$I = \left(\sum_i \Delta m_i\right) a^2 + 2\left(\sum_i \Delta m_i s_i\right) a + \sum_i \Delta m_i s_i^2 \qquad 8.4$$

In the first summand,

$$\sum_i \Delta m_i = M \qquad 8.5$$

is the total mass of the body. In the last summand,

$$\sum_i \Delta m_i s_i^2 = I_{CM} \qquad 8.6$$

is the moment of inertia of the body with respect to the centre-of-mass axis. In the middle summand,

$$\sum_i \Delta m_i s_i^2 = 0 \qquad 8.7$$

because the vectors s_i start from the axis through the centre of mass.

Thus Steiner's theorem follows from Eq. 8.4:

$$I^A = Ma^2 + I_{CM} \qquad 8.8$$

This theorem will be verified in the experiment with a flat circular disk as an example. Its moment of inertia I^A with respect to an axis of rotation at a distance a from the axis of symmetry is obtained from the period of oscillation T of a torsion axle to which the circular disk is attached. We have

$$I^A = D\left(\frac{T}{2\pi}\right)^2 \qquad 8.9$$

where D is restoring torque of the torsion axle.